A-Level

Mathematics

MM03 Mechanics 3

Final Mark scheme

6360

June 2017

Version/Stage: v1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	$\left[\frac{1}{2}\rho v^2\right] = \mathrm{ML^{-3}}\left(\mathrm{LT^{-1}}\right)^2$ $= \mathrm{ML^{-1}T^{-2}}$	M1		M1: Dimensions of $\frac{1}{2}\rho v^2$
	$[\rho gh] = \mathrm{ML^{-3}LT^{-2}L}$ $= \mathrm{ML^{-1}T^{-2}}$	M1		M1: Dimensions of ρgh
	$[P]$ and $[C] = \mathrm{MLT^{-2}L^{-2}}$ $= \mathrm{ML^{-1}T^{-2}}$	M1		M1: Dimensions of *P* and *C*
		A1		A1: All simplifications correct
	Hence, the equation is dimensionally consistent	E1	5	E1: Correct conclusion
	Total		**5**	

Q	Solution	Mark	Total	Comment
2 (a)	$x = 70\cos\theta t$	M1		M1: Correct expression for horizontal dis.
	$t = \dfrac{x}{70\cos\theta}$	A1		A1: Correct expression for *t*
	$y = 70\sin\theta t - \frac{1}{2}gt^2$	M1		M1: Correct expression for vertical dis. Allow sign errors.
	$y = 70\sin\theta \times \dfrac{x}{70\cos\theta} - \dfrac{1}{2} \times 9.8\left(\dfrac{x}{70\cos\theta}\right)^2$	m1		m1: Elimination of *t*
	$y = x\tan\theta - \dfrac{4.9x^2\left(1+\tan^2\theta\right)}{4900}$			
	$y = x\tan\theta - \dfrac{x^2\left(1+\tan^2\theta\right)}{1000}$ **AG**	A1	5	A1: Correct result from correct working

(b)(i)	$-25 = 500\tan\theta - \frac{(500)^2(1+\tan^2\theta)}{1000}$	M1		M1: substituting -25 and 500 for *y* and *x* resp.
	$10\tan^2\theta - 20\tan\theta + 9 = 0$	A1		A1:Correct equation
	$\tan\theta = \frac{20 \pm \sqrt{(20)^2 - 4(10)(9)}}{2(10)}$	m1		m1: Solution by formula or completing the sq. or by calculator
	$\tan\theta = 1.32\ ,\ \ 0.684$	A1		A1: Correct $\tan\theta$ values PI by correct θ values
	$\theta = 52.8^\circ\ ,\ \ 34.4^\circ$	A1	5	A1: Correct angles to 3 sf
(ii)	$t = \frac{500}{70\cos 34.4^\circ}$	M1		M1:Substituting correct value for x and their value for θ into their expression for *t* (from part (a))
	$t = 8.65$ s	A1	2	A1: AWRT 8.65 or 8.66
	Total		**12**	

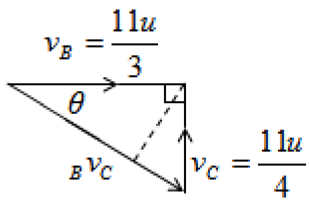
Q	Solution	Mark	Total	Comment
3(a)	$I=\int_0^4 (10-2t)\,\mathrm{d}t$	M1		M1: Integral with or without limits
	$I=\left[10t-t^2\right]_0^4$	A1		A1: Correct integration
	$I=24 \quad \text{Ns}$	A1	3	CAO
(b)	$24=2v-2(3)$	M1		M1: Impulse-momentum equation
	$v=15 \quad \text{ms}^{-1}$	A1F	2	FT their answer from (a)
c)	$\int_0^T (10-2t)\,\mathrm{d}t=2(11)-2(3)$	M1		M1:Correct impulse-momentum equation
	$10T-T^2=16$			
	$T^2-10T+16=0$	A1		A1:Correct quadratic equation
	$(T-8)(T-2)=0$	m1		m1: Factorisation or use of the formula, or use of calculator
	$T=2$ and 8 seconds	A1	4	CAO
	Total		**9**	

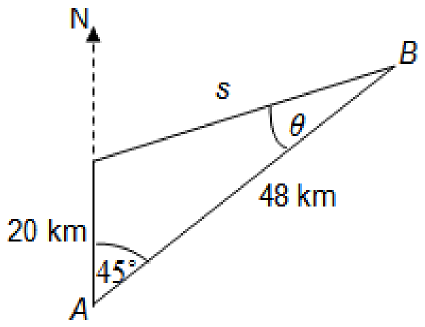
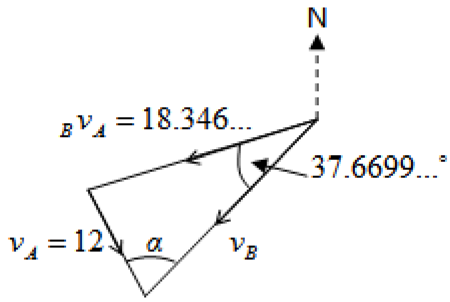
Q	Solution	Mark	Total	Comment
4(a)	CLM : $3mu\cos 30^\circ = 3mv_A\cos\alpha + 2mv_B$	M1		M1: CLM, allow sign error
		A1		A1: All correct
	Restitution : $v_B - v_A\cos\alpha = \frac{2}{3}\times u\cos 30^\circ$	M1		M1: NEL, allow sign error
		A1		A1: All correct
	$u\sin 30^\circ = v_A\sin\alpha$	B1		B1:Perpendicular component unchanged
	$3u\cos 30^\circ = 3v_B - 2u\cos 30^\circ + 2v_B$			
	$v_B = u\cos 30^\circ$	A1		A1: correct expression for v_B
	$3u\cos 30^\circ = 3v_A\cos\alpha + 2u\cos 30^\circ$			
	$3v_A\cos\alpha = u\cos 30^\circ$	A1		A1: correct expression for v_A
	$\frac{v_A\sin\alpha}{3v_A\cos\alpha} = \frac{u\sin 30^\circ}{u\cos 30^\circ}$	m1		m1:Forming an equation to find α
	$\tan\alpha = 3\tan 30^\circ$			
	$\alpha = 60^\circ$	A1	9	A1: CAO
(b)	Impulse = $2m\times u\cos 30^\circ$ OE	M1		M1: Impulse momentum equation
	$=\sqrt{3}mu$ or $1.73mu$ Ns	A1	2	A1: CAO
	Total		**11**	

Q	Solution	Mark	Total	Comment
5(a)	$y = ut\sin\theta - \frac{1}{2}gt^2\cos\alpha$	M1		M1: Expression for perpendicular height above the plane.
	$0 = ut\sin\theta - \frac{1}{2}gt^2\cos\alpha$	A1		A1: Correct expression with y=0
	$t = \frac{2u\sin\theta}{g\cos\alpha}$	A1		A1: Correct t
	$\dot{x} = u\cos\theta - gt\sin\alpha$	M1		M1: Expression for velocity component along the plane
	$0 = u\cos\theta - gt\sin\alpha$	A1		A1: Correct expression with $\dot{x} = 0$
	$t = \frac{u\cos\theta}{g\sin\alpha}$	A1		A1: Correct t
	$\frac{2u\sin\theta}{g\cos\alpha} = \frac{u\cos\theta}{g\sin\alpha}$ OE	m1		m1:Setting the two times equal
	$\frac{1}{\tan\alpha} = 2\tan\theta$			
	$\frac{3}{2} = 2\tan\theta$			
	$\tan\theta = \frac{3}{4}$ **AG**	A1	8	A1: CSO (Must see clear substitution for tanα

(b)	$x = ut\cos\theta - \frac{1}{2}gt^2\sin\alpha$	M1		M1:Correct expression for x, allow sign error
	$u\cos\theta = gt\sin\alpha$			
	$20 = gt^2\sin\alpha - \frac{1}{2}gt^2\sin\alpha$ OE	m1		m1:Substituting 20 for x and substituting for $u\cos\theta$
	$t^2 = \frac{40}{g\sin\alpha}$	A1		A1:Correct expression for t^2
	$t^2 = \frac{40}{9.8\sin 33.69^\circ}$ OE			
	$t = 2.7126... = 2.71$ s **AG**	A1	4	A1:CAO
(c)	$u = \frac{tg\sin\alpha}{\cos\theta}$	M1		M1:Correct expression for u
	$= \frac{2.7126...\times 9.8\times\sin 33.69^\circ}{\cos 36.87^\circ}$ OE	m1		m1:Substituting
	$= 18.4$ ms^{-1}	A1	3	A1: CAO, AWRT 18.4
	Total		**15**	

Q	Solution	Mark	Total	Comment
6(a)	$m(5u) + 2m(2u) = mv_A + 2mv_B$	M1		M1: Four non-zero momentum terms
		A1		A1: All correct
	$v_B - v_A = \frac{2}{3}(5u - 2u)$	M1		M1: NLR, allow sign errors
		A1		A1: Correct equation
	$v_A + 2v_B = 9u$			
	$v_B - v_A = 2u$			
	$3v_B = 11u$			
	$v_B = \frac{11u}{3}$ **AG**	A1		A1:Correct v_B from correct working
	$v_A = \frac{11u}{3} - 2u$			
	$v_A = \frac{5u}{3}$ **AG**	A1	6	A1:Correct v_A from correct working

(b)		B1		B1: Velocity triangle, PI from later work
	$\tan\theta = \dfrac{11u/4}{11u/3}$ OE	M1		M1:tanθ obtained from velocities
	$\tan\theta = \dfrac{3}{4}$			
	$\sin\theta = \dfrac{3}{5}$	A1		A1:Correct $\sin\theta$
	$d = s\sin\theta$	m1		m1:Identifying the shortest distance
	$d = s \times \dfrac{3}{5}$ or $\dfrac{3s}{5}$	A1	5	A1:CAO
	Total		**11**	
(b)	**Alternative:** Let D be the distance between *B* and *C* at time *t*. $D^2 = \left(s - \dfrac{11u}{3}t\right)^2 + \left(\dfrac{11u}{4}t\right)^2$	B1		B1:Correct expression for D^2
	$\dfrac{dD^2}{dt} = 2\left(s - \dfrac{11u}{3}t\right)\left(-\dfrac{11u}{3}\right) +$ $2\left(\dfrac{11u}{4}t\right)\dfrac{11u}{4}$			
	$2\left(s - \dfrac{11u}{3}t\right)\left(-\dfrac{11u}{3}\right) + 2\left(\dfrac{11u}{4}t\right)\dfrac{11u}{4} = 0$	M1		M1:Finding the time for shortest distance by calculus or completing the square
	$t = \dfrac{s\left(\dfrac{11u}{3}\right)}{\left(\dfrac{11u}{4}\right)^2 + \left(\dfrac{11u}{3}\right)^2}$ or $t = \dfrac{48s}{275u}$	A1		A1:Correct time
	$D^2 = \left(s - \dfrac{11u}{3} \times \dfrac{48s}{275u}\right) + \left(\dfrac{11u}{4} \times \dfrac{48s}{275u}\right)$	m1		m1:Finding d with their *t*
	$D^2 = \dfrac{225s^2}{625}$			
	$D = \dfrac{3s}{5}$	A1		A1: CAO

Q	Solution	Mark	Total	Comment
7(a)		B1		B1: Diagram, PI by correct method
	$s^2 = 20^2 + 48^2 - 2\times 20\times 48\cos 45^\circ$ $s = 36.6927...$ km	M1		M1: Cosine rule to find the relative distance
	${}_Bv_A = \dfrac{36.6927...}{2}$	m1		m1:Dividing their distance by 2
	$= 18.346...$ km h^{-1}	A1		A1:AWRT 18.3 or 18.4
	$\dfrac{\sin\theta}{20} = \dfrac{\sin 45}{36.6927...}$ $\theta = 22.669....^\circ$	M1		M1: Sine rule to find θ
	Bearing: 248°	A1	6	A1:AWRT 248°
(b)(i)		B1		B1: Diagram, PI by correct method
	$\dfrac{\sin\alpha}{18.346...} = \dfrac{\sin 37.6699...^\circ}{12}$	M1		M1: Sine rule to find α FT from (a)
	$\alpha = 69.1161...^\circ$, $110.8838...^\circ$	A1		A1:Correct values for α (PI by correct bearings)

	Bearings: $210^\circ - 69.1161...^\circ$ or $210^\circ - 69.1161...^\circ$ $= 141^\circ$ or 099°	A1	4	A1:Correct bearings , CAO
(ii)	$\dfrac{v_B}{\sin\left(180^\circ - 37.6699...^\circ - 69.1161...^\circ\right)} = \dfrac{12}{\sin 37.6699...^\circ}$ OE $v_B = 18.8$ km h^{-1}	M1 A1	 2	M1: Sine rule to find v_B, (FT angle from (b)(i)) A1: AWRT 18.8 km h^{-1}, CAO
	Total		**12**	